Int. J. Heat Mass Transfer. Vol. 10, pp. 1677–1692. Pergamon Press Ltd. 1967. Printed in Great Britain

EFFECTS OF VARIABLE PHYSICAL PROPERTIES ON LAMINAR FILM CONDENSATION OF SATURATED STEAM ON A VERTICAL FLAT PLATE

G. POOTS and R. G. MILES
Department of Applied Mathematics, University of Hull

(*Received* 16 *January* 1967 *and in revised form* 25 *May* 1967)

Abstract—A theoretical study is made of steady two-dimensional laminar film condensation of pure saturated steam at 100°C and atmospheric pressure on an isothermal vertical wall. The variable physical properties of the liquid condensate are taken to be those of saturated water at the appropriate temperatures. On neglecting the effects of surface tension at the liquid–vapour interface the governing equations for the liquid and vapour phases are simplified, using boundary-layer approximations, and admit to a solution in terms of similarity variables. From numerical solutions of the resulting ordinary differential equations flow characteristics for the liquid and vapour phases are determined for the wall temperatures 0, 10, 40, 70 and 90°C. The related results on wall heat transfer and mass flow in the liquid phase are presented and compared in detail with those from existing theories.

NOMENCLATURE

A_x,	$= g\rho(\rho - \rho_s^*)x^3/\mu^2$, a dimensionless group;
c_p,	specific heat;
F, G,	dimensionless velocity and thermal functions;
g,	acceleration due to gravity;
h,	local heat-transfer coefficient;
h_{fg},	latent heat of condensation;
k,	thermal conductivity;
K,	$= h_{fg}/c_p(T_s - T_w)$, a dimensionless group;
p,	pressure;
Pr,	$= \frac{c_p\mu}{k}$, the Prandtl number;
Re,	$= 4\Gamma/\mu$, film Reynolds number;
T,	temperature;
v_1, v_2,	condensate velocity components in the x_1, x_2 directions;
v_1^*, v_2^*,	vapour velocity components in the x_1^*, x_2^* directions;
x_1, x_2,	Cartesian coordinates in liquid phase;
x_1^*, x_2^*,	intrinsic coordinates in vapour phase.

Greek symbols

δ,	local thickness of condensate film;
ρ,	density;
μ,	viscosity;
Θ,	$= \frac{T - T_w}{T_s - T_w}$, dimensionless temperature;
Γ,	condensation rate.

Subscripts

s,	saturated;
w,	wall;
r,	reference;
D,	Drew reference;
M–S,	Minkowycz and Sparrow reference.

Superscripts

$*$,	vapour phase;
N,	Nusselt;
R,	Rohsenow;
C,	Chen.

INTRODUCTION

THE MECHANISM of laminar film condensation on a vertical isothermal flat plate maintained at constant temperature below the saturation temperature of the surrounding quiescent vapour was first studied by Nusselt [1]. Nusselt's theory consisted of simple force and heat balances in the condensate film and ignored effects due to inertia forces, thermal convection, interfacial shear and the dependence of the physical properties of the condensate on temperature. The effects of thermal convection were first investigated by Bromley [2] and Rohsenow [3]. To include the effects of thermal convection and inertia forces in the liquid film a new approach was formulated by Sparrow and Gregg [4] using boundary-layer approximations. This approach has been extended by Koh, Sparrow and Hartnett [5] to include the induced motion of the vapour and hence the effect of interfacial shear. In both [4] and [5] the boundary-layer type equations, as indicated by the work of Nusselt [1], yielded solutions in terms of similarity variables; the relevant ordinary differential equations were solved using numerical methods. Using integral boundary-layer techniques, Chen [6] has considered analytically the effects of thermal convection, inertia forces and interfacial shear.

In the foregoing analyses [1–6] the physical properties of the condensate are assumed to be independent of the temperature. Drew (see McAdams [7]) has shown that if, as in Nusselt's theory, the temperature distribution in the condensate is linear and on assuming that the condensate viscosity varies inversely with the temperature then the effects of variable viscosity on flow and heat transfer can be estimated by using the results of Nusselt's constant fluid property model with the viscosity evaluated at the reference temperature $T_D = T_w + 0{\cdot}25\,(T_s - T_w)$. Taking the above viscosity variation and on further assuming that the condensate conductivity varies linearly with the temperature Voskresenskiy [8] has re-investigated the model proposed by Nusselt. In a later paper Labuntsov [9] has refined this work, and shown that the flow and heat-transfer characteristics may be evaluated from the existing Nusselt formula, with condensate property values evaluated at the saturation temperature, provided a simple correction factor is used. Obviously if the conductivity of the condensate is assumed constant the modifications to Nusselt's model proposed by Drew and Voskresenskiy, respectively, would be identical. In a recent paper Minkowycz and Sparrow [10] have used Voskresenskiy's theory for the condensate film in an investigation of the effect of air as a non-condensable gas in the condensation of steam; other effects due to interfacial resistance or temperature jump at the liquid–vapour interface, superheating, and to variable fluid properties in the steam–air mixture were considered. The property values for water were taken from Eckert and Drake [11] and a new reference temperature $T_{M-S} = T_w + 0{\cdot}31(T_\delta - T_w)$ was derived; here T_δ is used to denote the bulk saturation temperature of the vapour. In particular for the condensation of pure saturated steam at atmospheric pressure and at 100°C the results obtained for wall heat transfer using the Nusselt theory and the Minkowycz and Sparrow reference temperature will be, essentially, equivalent to those obtained by employing the Labuntsov correction factor as described above.

It is clear that no general conclusions on the combined effects due to thermal convection, inertia forces, interfacial shear and variable physical properties can be drawn from the essentially separate constant fluid property solutions [2–6] and the relatively simple variable fluid property modifications [7–9]. Indeed all of these effects on flow and heat-transfer characteristics are intimately connected. The motive for the present investigation is to provide further information on the combined effects discussed in references [2–9].

The flow configuration to be studied is as follows. An isothermal vertical flat plate is inserted in a large volume of quiescent pure saturated steam at atmospheric pressure. If the plate temperature is T_w then provided $T_w < T_s$, the saturation temperature of the vapour, steady two-dimensional film condensation will occur on the plate. It is assumed that laminar flow is induced by gravity within the liquid and vapour phases. Surface tension effects at the liquid–vapour interface are neglected implying that the interface is smooth and free from waves.

BASIC GOVERNING EQUATIONS FOR THE LIQUID AND VAPOUR PHASES

Let x_1 measure the distance in the downward direction along the plate and x_2 the distance normal to the plate; the leading edge is located at $x_1 = x_2 = 0$. In the liquid film the velocity field is denoted by the components v_1 and v_2 in the x_1 and x_2 directions respectively, and the local temperature field by T. The liquid–vapour interface is defined by

$$x_2 = \delta(x_1). \tag{1}$$

Because the liquid film is thin all spatial derivatives across the film are large in comparison to those along the film. Consequently the rate of change of velocity or temperature across the film is large in comparison with corresponding changes along its length. The governing equations expressing conservation of mass, momentum and thermal energy in the liquid phase now simplify to the boundary-layer equations as follows:

$$\frac{\partial}{\partial x_1}(\rho v_1) + \frac{\partial}{\partial x_2}(\rho v_2) = 0, \tag{2}$$

$$\rho\left(v_1\frac{\partial v_1}{\partial x_1} + v_2\frac{\partial v_1}{\partial x_2}\right) = -\frac{\partial p}{\partial x_1} + \rho g + \frac{\partial}{\partial x_2}\left(\mu\frac{\partial v_1}{\partial x_2}\right), \tag{3}$$

$$0 = -\frac{\partial p}{\partial x_2}, \tag{4}$$

$$\rho c_p\left(v_1\frac{\partial T}{\partial x_1} + v_2\frac{\partial T}{\partial x_2}\right) = \frac{\partial}{\partial x_2}\left(k\frac{\partial T}{\partial x_2}\right), \tag{5}$$

where in the latter equation viscous dissipation and work done against compression have been neglected. These equations must be supplemented with certain fluid property relations. It is assumed for the condensate

$$\rho = \rho(T), \quad c_p = c_p(T), \quad \mu = \mu(T) \text{ and } k = k(T), \tag{6}$$

and moreover this data is available from experiment.

In the derivation of the relevant boundary-layer equations for the vapour phase it is convenient to choose a set of intrinsic coordinates attached to the interface. Let x_1^* measure the distance along the interface, x_2^* the distance along the normal and v_1^*, v_2^* denote the associated velocity components. The governing equations, expressing conservation of mass and momentum, in the vapour phase can be written as the boundary-layer equations:

$$\frac{\partial v_1^*}{\partial x_1^*} + \frac{\partial v_2^*}{\partial x_2^*} = 0, \tag{7}$$

$$\rho_s^*\left(v_1^*\frac{\partial v_1^*}{\partial x_1^*} + v_2^*\frac{\partial v_1^*}{\partial x_2^*}\right) = -\frac{\partial p^*}{\partial x_1^*} + \rho_s^* g + \mu_s^*\frac{\partial^2 v_1^*}{\partial x_2^{*2}} \tag{8}$$

and

$$0 = \frac{\partial p^*}{\partial x_2^*}. \tag{9}$$

In doing so it is assumed that the radius of curvature of the interface is large compared with the thickness of the film (see Goldstein [12]); moreover to the same approximation

$$\left.\begin{aligned} &x_1 \simeq x_1^* \\ &\text{and} \\ &x_2 = x_2^* + \delta(x_1). \end{aligned}\right\} \tag{10}$$

The physical boundary conditions are:

$$\left.\begin{aligned} &v_1 = v_2 = 0, \quad T = T_w \\ &\text{for } \; x_2 = 0, \quad x_1 \geqslant 0, \\ &T = T_s \quad \text{for } \; x_2 = \delta(x_1), \quad x_1 \geqslant 0, \end{aligned}\right\} \tag{11}$$

in the liquid phase, whilst at large distance from the plate in the vapour phase

$$v_1^* \to 0 \quad \text{as} \quad x_2 \to \infty, \quad x_1 \geqslant 0. \qquad (12)$$

It now remains to express conditions for the location of the liquid–vapour interface and relevant criteria for matching v, p in the liquid phase with v^*, p^* in the vapour phase at this location. Obviously four conditions are required, three for matching v_1, v_2 and p and one for deducing the form of (1). They are derived as in [5] from consideration of the velocities, mass flow, shear stresses and the heat flux vector at the interface. These conditions are now expressed within the frame-work of the boundary-layer approximations already used. At the interface the tangential components of the liquid and vapour phases must be continuous giving the condition:

$$v_1 = v_1^* \quad \text{for} \quad x_2 = \delta(x_1), \quad x_1 \geqslant 0. \qquad (13)$$

The local mass flux crossing the liquid–vapour interface is continuous, and hence:

$$\rho_s\left(v_2 - v_1\frac{d\delta}{dx_1}\right) = \rho_s^* v_2^* \quad \text{for} \quad x_2 = \delta(x_1), x_1 \geqslant 0. \qquad (14)$$

On considering the stress tensor at the interface yields the conditions:

$$p = p^*, \quad \mu_s\frac{\partial v_1}{\partial x_2} = \mu_s^*\frac{\partial v_1^*}{\partial x_2^*} \quad \text{for} \quad x_2 = \delta(x_1)\, x_1 \geqslant 0; \qquad (15)$$

the former condition is true only if surface tension is neglected, whilst the latter is valid even if surface tension is included. It now remains to derive the fourth condition for determining $\delta(x_1)$. This is obtained on applying the first law of thermodynamics to a control volume of the condensate bounded by the planes x_1, $x_1 + dx_1$, the wall $x_2 = 0$ and the appropriate section of the liquid–vapour interface. The condition is as follows:

$$-k_w\int_0^{x_1}\left(\frac{\partial T}{\partial x_2}\right)_w dx_1 + h_{fg}\int_0^{\delta(x_1)}\rho v_1\,dx_2 + \int_0^{\delta(x_1)}\rho v_1(c_{p_s}T_s - c_pT)\,dx_2 + \int_0^{x_1}\int_0^{\delta(x_1)}\rho T\frac{\partial c_p}{\partial T}\left(v_1\frac{\partial T}{\partial x_1} + v_2\frac{\partial T}{\partial x_2}\right)dx_1\,dx_2 = 0. \qquad (16)$$

The various terms occurring in this energy balance are readily identified. The first term is the heat transferred by conduction from the condensate to the plate; the second term is due to the latent heat of condensation; the third term is due to subcooling of the condensate below saturation temperature; the fourth term is essentially a dissipative effect associated with the convective rate of change of the thermal capacity. Within the frame-work of the boundary-layer hypothesis viscous dissipation and work done against compression are negligible; the work done by the surface tension at the interface has been ignored. Finally, by virtue of the interface condition $p = p^*$, and equations (4, 9) and the boundary condition (12) it follows, as is usual in boundary-layer theory, that the pressure along a normal to the wall in both phases is constant and $\partial p/\partial x_1 = \partial p^*/\partial x_1^* = \rho_S^* g$. This completes the formulation of the equations for laminar film condensation in which the effect of variable physical properties within the condensate has been taken into account.

It is convenient to introduce new dependent and independent variables as follows: for the liquid phase

$$\left.\begin{aligned} &x_1 = LX_1, \quad x_2 = LX_2, \quad \delta = L\Delta, \\ &v_1 = \frac{\nu_s}{L}V_1, \quad v_2 = \frac{\nu_s}{L}V_2, \\ &\Theta = \frac{T - T_w}{T_s - T_w}, \end{aligned}\right\} \qquad (17)$$

and for the vapour phase

$$x_1^* = LX_1^*, \qquad x_2^* = LX_2^*,$$

$$v_1 = \frac{v_s^*}{L} V_1^*, \qquad v_2^* = \frac{v_s^*}{L} \; V_2^*, \tag{18}$$

where the representative length $L = (v_s^2/g)^{\frac{1}{3}}$.

Upon introducing a stream function ψ defined by

$$\rho V_1 = \rho_s \frac{\partial \psi}{\partial X_2}, \qquad \rho V_2 = -\, \rho_s \frac{\partial \psi}{\partial X_1}, \tag{19}$$

the required similarity solution (see, for example, references [4, 5]) of the liquid film equations (1–6) is in the form:

$$\psi = (4X_1)^{\frac{3}{4}} F(\eta), \qquad \Theta = G(\eta),$$

$$\Delta = \phi(4X_1)^{\frac{1}{4}}, \tag{20}$$

where ϕ is a constant to be determined. Here η is the similarity variable defined by

$$\eta = \tilde{X}_2/(4X_1)^{\frac{1}{4}} \tag{21}$$

where $\tilde{X}_2$ is the Howarth–Dorodnitsyn variable:

$$\tilde{X}_2 = \int_0^{X_2} \frac{\rho}{\rho_s} \mathrm{d}X_2. \tag{22}$$

The velocity field is related to F by

$$v_1 = (\nu_s g)^{\frac{1}{3}} (4X_1)^{\frac{1}{2}} \mathrm{d}F/\mathrm{d}\eta,$$

$$v_2 = -\frac{\rho_s}{\rho} (\nu_s g)^{\frac{1}{3}} (4X_1)^{-\frac{1}{4}}$$

$$\left\{\left(3F - \eta \frac{\mathrm{d}F}{\mathrm{d}\eta}\right) + (4X_1)^{\frac{3}{4}} \frac{\mathrm{d}F}{\mathrm{d}\eta} \left(\frac{\partial \tilde{X}_2}{\partial X_1}\right)_{X_2}\right\}; \tag{23}$$

the physical thickness of the liquid film is determined by the expression

$$\delta = (\nu_s^2/g)^{\frac{1}{3}} (4X_1)^{\frac{1}{4}} \int_0^{\phi} \frac{\rho_s}{\rho} \mathrm{d}\eta. \tag{24}$$

Accordingly the liquid film equations (2–5) reduce to the ordinary differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{\rho\mu}{\rho_s\mu_s} \frac{\mathrm{d}^2F}{\mathrm{d}\eta^2}\right) + 3F \frac{\mathrm{d}^2F}{\mathrm{d}\eta^2} - 2\left(\frac{\mathrm{d}F}{\mathrm{d}\eta}\right)^2$$

$$+ \frac{\rho - \rho_s^*}{\rho} = 0, \tag{25}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{\rho k}{\rho_s k_s} \frac{\mathrm{d}G}{\mathrm{d}\eta}\right) + 3Pr_s \frac{c_p}{c_{p_s}} F \frac{\mathrm{d}G}{\mathrm{d}\eta} = 0. \tag{26}$$

The condensate property relations (6) take the simple form:

$$\left.\begin{aligned} &\rho = \rho(G;T_w,T_s), \qquad c_p = c_p(G;T_w,T_s), \\ &\mu = \mu(G;T_w,T_s) \quad \text{and} \quad k = h(G;T_w,T_s). \end{aligned}\right\} \tag{27}$$

Similarly, for the vapour phase, on introducing a stream function ψ^* defined by

$$V_1^* = \frac{\partial \psi^*}{\partial X_2^*}, \quad V_2^* = -\frac{\partial \psi^*}{\partial X_1^*}, \tag{28}$$

the corresponding similarity solution of equations (7–9) is in the form:

$$\psi^* = (4X_1^*)^{\frac{3}{4}} F^*(\eta^*) \tag{29}$$

where η^* is the vapour phase similarity variable defined by

$$\eta^* = X_2^*/(4X_1^*)^{\frac{1}{4}}. \tag{30}$$

The vapour velocity field is related to F^* by

$$v_1^* = \nu_s^* \; {\frac{\nu_s^2}{g}}^{-\frac{1}{3}} (4X_1^*)^{\frac{1}{2}} \mathrm{d}F^*/\mathrm{d}\eta^*, \tag{31}$$

$$\nu_s^* = -\,\nu_s^* \left(\frac{\nu_s^2}{g}\right)^{-\frac{1}{3}} (4X_1^*)^{-\frac{1}{4}} (3F^*$$

$$- \eta^* \mathrm{d}F^*/\mathrm{d}\eta^*). \tag{32}$$

The vapour phase equations (7–9) reduce to the following ordinary differential equation:

$$\frac{\mathrm{d}^3F^*}{\mathrm{d}\eta^{*3}} + 3F^* \frac{\mathrm{d}^2F^*}{\mathrm{d}\eta^{*2}} - 2 \; {\frac{\mathrm{d}F^*}{\mathrm{d}\eta^*}}^{2} = 0. \tag{33}$$

In the new variables η and η^* the location of the liquid–vapour interface at any station $X_1 = X_1^*$ is given by $\eta = \phi$ in the liquid phase and $\eta^* = 0$ in the vapour phase. The physical boundary conditions (11) and (12) together with the interfacial matching conditions (13–15) are equivalent to

$$F = \mathrm{d}F/\mathrm{d}\eta = G = 0 \quad \text{at} \quad \eta = 0; \qquad (34)$$

$$G = 1, \mathrm{d}F/\mathrm{d}\eta = \frac{\nu_s^*}{\nu_s}\mathrm{d}F^*/\mathrm{d}\eta^*,$$

$$F = \frac{\rho_s^*\nu_s^*}{\rho_s\nu_s}F^*, \qquad \mathrm{d}^2F/\mathrm{d}\eta^2$$

$$= \frac{\nu_s^*\mu_s^*}{\nu_s\mu_s}\mathrm{d}^2F^*/\mathrm{d}\eta^{*2}, \qquad \text{at} \quad \eta = \phi; \qquad (35)$$

$$\mathrm{d}F^*/\mathrm{d}\eta^* \to 0 \text{ at } \eta^* \to \infty; \qquad (36)$$

and finally the heat balance transforms to

$$\frac{\rho_w k_w}{3\rho_s k_s Pr_s}\left(\frac{\mathrm{d}G}{\mathrm{d}\eta}\right)_{\eta=0} - \frac{h_{fg}}{c_p(T_s - T_w)}F(\phi)$$

$$+ \int_0^{\phi}\left\{\left[F\left(G + \frac{T_w}{(T_s - T_w)}\right)\frac{\partial}{\partial G}\left(\frac{c_p}{c_{p_s}}\right)\right]\frac{\mathrm{d}G}{\mathrm{d}\eta}\right.$$

$$\left. + \left[\frac{c_p}{c_{p_s}}\left(G + \frac{T_w}{(T_s - T_w)}\right) - \frac{T_s}{(T_s - T_w)}\right]\frac{\mathrm{d}F}{\mathrm{d}\eta}\right\}\mathrm{d}\eta$$

$$= 0. \qquad (37)$$

Thus the problem of laminar film condensation on a vertical flat plate has been reduced to to the solution of an eighth-order, three-point, boundary-value problem defined by the differential equations (25, 26, 33) subject to the boundary conditions (34–37). The next section briefly describes the numerical procedures used in obtaining solutions for the condensation of steam at 100°C and atmospheric pressure.

NUMERICAL SOLUTIONS FOR THE CONDENSATION OF STEAM

Property values

For the condensate the physical properties are taken to be those of saturated water at the appropriate temperatures. The data for saturated water are taken from tables compiled by Mayhew and Rogers [13]. In cgs units these experimental values are adequately represented, as is discussed in [14], by algebraic expressions in the form:

$$\left.\begin{aligned} \rho &= \sum_{i=0}^{10} A_i\theta^i, & c_p &= \sum_{i=0}^{10} B_i\theta^i, \\ \mu &= \exp\left(\sum_{i=0}^{10} C_i\theta^i\right), & k &= \exp\left(\sum_{i=0}^{10} D_i\theta^i\right), \end{aligned}\right\} \quad (38)$$

where $\theta = (T - 50°\mathrm{C})/50°\mathrm{C}$; the numerical values of the A_i, B_i, C_i and D_i coefficients are listed in [14]. Data is required for the gradient $\partial c_p/\partial T$. The experimental data for c_p is first smoothed using localized quadratic smoothing formulae (see Buckingham [15]) and acceptable values of $\partial c_p/\partial T$ obtained for $0 \leqslant T \leqslant 100°\mathrm{C}$. The method of least squares gave the "best fit" for these values as

$$\partial c_p/\partial T = \sum_{i=0}^{3} E_i[(T - 30°\mathrm{C})/5°\mathrm{C}]^i$$

$$\text{for} \quad 0 \leqslant T \leqslant 100°\mathrm{C}; \qquad (39)$$

for saturated water the coefficients E_i are:

$$E_0 = -2{\cdot}60 \times 10^{-5}, \qquad E_1 = 4{\cdot}18 \times 10^{-5},$$

$$E_2 = -0{\cdot}63 \times 10^{-5} \text{ and } E_3 = 4{\cdot}03 \times 10^{-7}.$$

The property values ρ_s^*, $c_{p_s}^*$, μ_s^*, k_s^*, h_{fg} for steam at 100°C and atmospheric pressure were taken from [13].

For convenience some specimen values of the physical properties used in the calculations are listed in Table 1.

Solutions

Numerical solutions for the laminar film condensation of steam have been obtained for $T_w = 0$, 10, 40, 70 and 90°C. The integrations of the differential equations (25, 26, 33) subject to the boundary conditions (34–36) were performed using Gill's modification of the Runge–Kutta method. The scheme adopted for the numerical solution of the three-point boundary-value problem is as follows. Given T_w, estimates of ϕ and the initial values $(\mathrm{d}^2F/\mathrm{d}\eta^2)_{\eta=0}$ and $(\mathrm{d}G/\mathrm{d}\eta)_{\eta=0}$ are obtained using the constant fluid property model of Nusselt [1]. For this value of ϕ the system of equations are solved

Table 1

Phase	Water		Steam
	$T = 0°\text{C}$	$T_s = 100°\text{C}$	$T_s^* = 100°\text{C}$
ρ, (g/cm^3)	0·9998	0·9578	$0{\cdot}5977 \times 10^{-3}$
c_p, (cal/g degC)	1·0055	1·0075	
μ, (g/cm s)	$1{\cdot}782 \times 10^{-2}$	$2{\cdot}812 \times 10^{-3}$	$1{\cdot}245 \times 10^{-4}$
ν, (cm^2/s)	$1{\cdot}782 \times 10^{-2}$	$2{\cdot}936 \times 10^{-3}$	$2{\cdot}083 \times 10^{-1}$
k, (cal/s cm degC)	$1{\cdot}316 \times 10^{-3}$	$1{\cdot}631 \times 10^{-3}$	$5{\cdot}735 \times 10^{-5}$
Pr	13·616	1·7370	
h_{fg}, (cal/g)			538·83

iteratively. The left-hand side of equation (37) is then computed. The estimate of ϕ is slightly perturbed and the iterative process repeated, ending with a second estimate for the left-hand side of (37). A new estimate of ϕ is obtained and the complete process repeated until the heat balance (37) and the vapour phase condition (36) are simultaneously satisfied. During the course of the numerical work it was established that the dissipative term involving $\partial c_p/\partial T$ occurring in the heat balance was negligibly small.

The unknown characteristics $(\mathrm{d}^2F/\mathrm{d}\eta^2)_{\eta=0}$, $(\mathrm{d}G/\mathrm{d}\eta)_{\eta=0}$, ϕ, and $F(\phi)$ have been rounded off to four decimals and are listed in Table 2. In Table 3 the velocity functions F and F^* and the temperature function G are given for $T_w = 0°\text{C}$ and $T_s = 100°\text{C}$.

SUMMARY OF KNOWN THEORETICAL RESULTS ON FLOW AND HEAT TRANSFER

Before presenting a general discussion of the numerical results it is convenient at this stage to give definitions of certain flow and heat-transfer characteristics and the relevant relations obtained by other workers. The most important results of practical interest are the mass flow and heat transfer. The mass flow or condensation rate is defined as

$$\Gamma_{x_1} = \int_0^\delta \rho v_1 \,\mathrm{d}x_2. \tag{40}$$

The local wall heat-transfer coefficient h_{x_1} and Nusselt number Nu_{x_1} are defined as

$$h_{x_1} = \frac{q}{T_s - T_w} = \frac{(k\partial T/\partial x_2)_{x_2=0}}{T_s - T_w},$$

$$Nu_{x_1} = \frac{x_1 h_{x_1}}{k}; \tag{41}$$

for a section of plate, length l, the average wall heat-transfer coefficient h and average Nusselt number are then defined by

$$h = \frac{1}{l}\int_0^l h_{x_1}\,\mathrm{d}x_1, \qquad Nu = \frac{lh}{k}. \tag{42}$$

Table 2

$T_s - T_w$	$(\mathrm{d}^2F/\mathrm{d}\eta^2)_{\eta=0}$	$(\mathrm{d}G/\mathrm{d}\eta)_{\eta=0}$	ϕ	$F(\phi)$	$\int_0^\phi \frac{\rho_s}{\rho}\mathrm{d}\eta$
100	0·1104	1·5439	0·7485	0·0440	0·7284
90	0·1423	1·5831	0·7070	0·0426	0·6891
60	0·2378	1·7803	0·5859	0·0354	0·5747
30	0·3054	2·2148	0·4559	0·0233	0·4511
10	0·2910	3·0400	0·3303	0·0109	0·3295

Table 3

Phase	Water ($T_w = 0°C$)					Steam ($T_s = 100°C$)			
η/ϕ	$F \times 10$	$\left(\frac{dF}{d\eta}\right) \times 10$	$\left(\frac{d^2F}{d\eta^2}\right) \times 10$	G	$\left(\frac{dG}{d\eta}\right)$	η^*	F^*	$\left(\frac{dF^*}{d\eta^*}\right) \times 10^3$	$\left(\frac{d^2F^*}{d\eta^{*2}}\right) \times 10^2$
0·00	0·0000	0·0000	1·1041	0·0000	1·5439	0·0	0·9944$_1$	1·4698	−0·4399
0·05	0·0008	0·0442	1·2582	0·0570	1·5052	0·1	0·9945$_4$	1·0895	−0·3264
0·10	0·0034	0·0940$_5$	1·4027	0·1127	1·4716	0·2	0·9946$_3$	0·8073	−0·2422
0·15	0·0079	0·1490	1·5325	0·1671$_5$	1·4392	0·3	0·9947$_0$	0·5979	−0·1797
0·20	0·0146	0·2085	1·6447	0·2205	1·4122	0·4	0·9947$_5$	0·4425	−0·1333
0·25	0·0236	0·2719	1·7375	0·2729	1·3905	0·5	0·9947$_9$	0·3272	−0·0989
0·30	0·0350	0·3383	1·8095	0·3246	1·3723	0·6	0·9948$_2$	0·2417	−0·0734
0·35	0·0489	0·4070	1·8589	0·3756$_5$	1·3558	0·7	0·9948$_3$	0·1782	−0·0545
0·40	0·0655	0·4771	1·8840	0·4261	1·3405	0·8	0·9948$_5$	0·1312	−0·0404
0·45	0·0856	0·5477	1·8833$_5$	0·4760	1·3267	0·9	0·9948$_7$	0·0962	−0·0299
0·50	0·1064$_5$	0·6178	1·8558	0·5254	1·3146	1·0	0·9948$_7$	0·0703	−0·0222
0·55	0·1308$_5$	0·6863	1·8003	0·5744	1·3039	1·1	0·9948$_8$	0·0511	−0·0165
0·60	0·1578	0·7522	1·7156	0·6230	1·2941	1·2	0·9948$_8$	0·0368	−0·0122$_5$
0·65	0·1871	0·8143	1·6001	0·6713	1·2848	1·3	0·9948$_9$	0·0262	−0·0091
0·70	0·2187	0·8716	1·4540	0·7192	1·2760	1·4	0·9948$_9$	0·0183$_5$	−0·0067
0·75	0·2523	0·9227	1·2747	0·7668	1·2682	1·5	0·9948$_9$	0·0125	−0·0050
0·80	0·2876	0·9665$_5$	1·0620	0·8141	1·2614	1·6		0·0082	−0·0037
0·85	0·3245	1·0018	0·8153	0·8612	1·2537	1·8		0·0026	−0·0020
0·90	0·3625	1·0271	0·5337	0·9079	1·2422	2·0		0·0000	−0·0011
0·95	0·4012	1·0412	0·2160	0·9541	1·2275	2·2			−0·0006
1·00	0·4403	1·0428	−0·1382	1·0000	1·2278	2·4			−0·0003

Constant fluid property model investigations

The original theory according to Nusselt [1] involved the solutions of equations (3) and (5) subject to the boundary conditions:

$$v_1(0) = 0, \qquad T(0) = T_w, \qquad \left(\frac{\partial v_1}{\partial x_2}\right)_{x_2=\delta} = 0,$$
$$T(\delta) = T_s, \tag{43}$$

together with the simplified heat balance:

$$k\left(\frac{\partial T}{\partial x_2}\right)_{x_2=0} = h_{fg}\frac{d\Gamma_{x_1}}{dx_1}. \tag{44}$$

Terms involving inertia forces and convective heat transfer were neglected as well as the effect of interfacial shear. Nusselt obtained the following results:

$$\frac{v_1}{(4x_1g)^{\frac{1}{2}}} = \left(1 - \frac{\rho^*}{\rho}\right)^{\frac{1}{2}}(Pr\,K)^{-\frac{1}{4}}\left[\frac{x_2}{\delta_{x_1}} - \frac{1}{2}\left(\frac{x_2}{\delta_{x_1}}\right)^2\right],$$
$$T = T_w + (T_s - T_w)\frac{x_2}{\delta_{x_1}}; \tag{45}$$

the local film thickness of the condensate layer is:

$$\delta_{x_1} = (\surd 2)x_1\,(A_{x_1}Pr\,K)^{-\frac{1}{4}}, \tag{46a}$$

where $A_{x_1} = g\rho(\rho - \rho_s^*)x_1^3/\mu^2$ is the ratio of the gravitational buoyancy force to the viscous force, $Pr = c_p\mu/k$ is the Prandtl number and $K = h_{fg}/c_p(T_s - T_w)$ is the ratio of the latent heat of the vapour to condensate enthalpy change†. For the condensation rate and heat transfer Nusselt's theory yields

$$\Gamma_{x_1} = \tfrac{1}{3}\mu A_{x_1}(\delta_{x_1}/x_1)^3, \tag{46b}$$

$$h_{x_1} = k(A_{x_1}Pr\,K)^{\frac{1}{4}}/(x_1\surd 2), \tag{46c}$$

$$Nu_{x_1} = (A_{x_1}Pr\,K)^{\frac{1}{4}}/\surd 2, \tag{46d}$$

and finally

$$h = \tfrac{4}{3}h_l, \qquad Nu = \tfrac{4}{3}Nu_l. \tag{46e}$$

On including the effect of thermal convection,

---

† In analogy with the dynamically similar free convection flow on a cooled vertical wall the dimensionless group A_{x_1} is effectively a condensate Grashof number.

Rohsenow [3] obtained the heat-transfer expression

$$h^R_{x_1} = (1 + 0{\cdot}68/K)^{\frac{1}{4}}\, h^N_{x_1}; \qquad (47)$$

the superscripts R and N denote the values obtained by Rosenhow and Nusselt respectively.

Further investigation by Chen [6], to include the effects of inertia forces, thermal convection and interfacial shear, yielded the results:

$$h^C_{x_1} = h^N_{x_1}\left\{\frac{1 + 0{\cdot}68\dfrac{1}{K} + 0{\cdot}02\dfrac{1}{Pr\,K^2}}{1 + 0{\cdot}85\dfrac{1}{Pr\,K} - 0{\cdot}15\dfrac{1}{Pr\,K^2}}\right\}^{\frac{1}{4}}, \qquad (48a)$$

and

$$\Gamma^C_{x_1} = \Gamma^N_{x_1}\frac{h^C_{x_1}}{h^N_{x_1}}\left\{1 + 0{\cdot}375\frac{1}{K} + 0{\cdot}020\frac{1}{Pr\,K^2} - 0{\cdot}0005\frac{1}{Pr^2\,K^3}\right\}. \qquad (48b)$$

Relations (48a) and (48b) agree, to within one per cent, with the boundary-layer calculations of Koh, Sparrow and Hartnett [5] when $Pr \geqslant 1$ and $K \geqslant 5$.

Variable fluid property model investigations

To date the effect of variations in the physical properties of the condensate have been considered in relation to Nusselt's model [1]. These improvements, due to Drew [7], Voskresenskiy [8], Labuntsov [9] and Minkowycz and Sparrow [10], will now be considered in some detail since they are particularly relevant to the results given in the present paper.

On neglecting the effects of inertia forces, convection transfer and interfacial shear, the governing equations of motion (1–5) for the condensate are:

$$\frac{\partial}{\partial x_2}\left(\mu\frac{\partial v_1}{\partial x_2}\right) = -(\rho - \rho^*_s)\,g, \qquad (49)$$

$$\frac{\partial}{\partial x_2}\left(k\frac{\partial T}{\partial x_2}\right) = 0, \qquad (50)$$

and by virtue of equation (50) the heat balance becomes

$$h_{fg}\frac{\mathrm{d}\Gamma_{x_1}}{\mathrm{d}x_1} = k\frac{\partial T}{\partial x_2} = \text{const.} \qquad (51)$$

The boundary conditions are given by equation (43) and the fluid property relations by (6). Note that c_p does not appear in this model since the convective heat-transfer term in (50) is omitted. The liquid-phase similarity variables [see equations (17) and (20)] are introduced as follows:

$$x_1 = LX_1, \qquad x_2 = LX_2,$$

$$\delta = L(4X_1)^{\frac{1}{4}}\tilde{\phi}, \qquad v_1 = \frac{v_s}{L}(4X_1)^{\frac{1}{2}} f(\tilde{\eta}),$$

$$g(\tilde{\eta}) = \frac{T - T_w}{T_s - T_w}, \qquad \tilde{\eta} = X_2/(4X_1)^{\frac{1}{4}},$$

$$L = \left(\frac{v_s^2}{g}\right)^{\frac{1}{3}}. \qquad (52)$$

Consequently equations (49–51) and the property relations (6) simplify to

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{\eta}}\left(\frac{\mu}{\mu_s}\frac{\mathrm{d}f}{\mathrm{d}\tilde{\eta}}\right) = -\frac{\rho - \rho_s}{\rho_s}, \qquad f(0) = \left(\frac{\mathrm{d}f}{\mathrm{d}\tilde{\eta}}\right)_{\tilde{\phi}} = 0, \qquad (53)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{\eta}}\left(\frac{k}{k_s}\frac{\mathrm{d}g}{\mathrm{d}\tilde{\eta}}\right) = 0, \qquad g(0) = 0,\, g(\tilde{\phi}) = 1, \qquad (54)$$

$$3Pr_sK_s\int_0^{\tilde{\phi}}\frac{\rho}{\rho_s}f\,\mathrm{d}\tilde{\eta} = \frac{k}{k_s}\frac{\mathrm{d}g}{\mathrm{d}\tilde{\eta}} = \text{const.}, \qquad (55)$$

and

$$\rho = \rho(g;\, T_w, T_s), \qquad \mu = \mu(g;\, T_w, T_s) \quad \text{and} \quad k = k(g;\, T_w, T_s), \qquad (56)$$

respectively.

It is convenient to choose the dimensionless temperature g, instead of $\tilde{\eta}$, as the independent variable (see Carslaw and Jaeger [16]. For, from equation (54),

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{\eta}} \equiv \frac{k_sE}{k}\frac{\mathrm{d}}{\mathrm{d}g}, \qquad (57)$$

where the constant E is

$$E = \frac{1}{\tilde{\phi}}\int_0^1 \frac{k}{k_s}\,\mathrm{d}g, \tag{58}$$

and so equation (53) becomes:

$$\frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{k_s\mu}{k\mu_s}\frac{\mathrm{d}f}{\mathrm{d}g}\right) = -\frac{\rho - \rho_s^*}{\rho_s}\frac{k}{E^2 k_s} \tag{59}$$

subject to the boundary conditions

$$f = 0 \text{ at } g = 0 \quad \text{and} \quad \mathrm{d}f/\mathrm{d}g = 0 \text{ at } g = 1; \tag{60}$$

the heat balance (55) transforms to:

$$\int_0^1 \frac{k\rho}{k_s\rho_s} f\,\mathrm{d}g = E^2/3Pr_s K_s. \tag{61}$$

The differential equation (59) subject to (60) is readily integrated, and on using (58) and (61) flow and heat-transfer characteristics can be evaluated.

The condensate velocity is given by

$$v_1\left(\frac{Pr_s K_s}{4x_1 g}\right)^{\frac{1}{2}} \int\limits_{g_2=0}^{g_2=g} \frac{k\mu_s}{k_s\mu}\left[\int\limits_{g_1=g_2}^{g_2=1} \frac{k}{k_s}\left(\frac{\rho - \rho^*}{\rho_s}\right)\mathrm{d}g_1\right] \times \mathrm{d}g_2/\epsilon(T_w;T_s)^2; \tag{62}$$

the flow and heat-transfer characteristics are:

$$\frac{\delta_{x_1}}{(\delta_{x_1}^N)_s} = \left(1 - \frac{\rho_s{}^*}{\rho_s}\right)^{\frac{1}{4}} \int_0^1 \frac{k}{k_s}\,\mathrm{d}g \Big/ \epsilon(T_w;T_s), \tag{63}$$

$$\frac{\Gamma_{x_1}}{(\Gamma_{x_1}^N)_s} = \frac{h_{x_1}}{(h_{x_1}^N)_s} = \left(1 - \frac{\rho_s{}^*}{\rho_s}\right)^{-\frac{1}{4}} \epsilon(T_w;T_s); \tag{64}$$

the factor $\epsilon(T_w; T_s)$ is given by

$$\epsilon(T_w;T_s) = \left\{3\int\limits_{g_3=0}^{g_3=1} \frac{k\rho}{k_s\rho_s}\left[\int\limits_{g_2=0}^{g_2=g_3}\left(\frac{k\mu_s}{k_s\mu}\int\limits_{g_1=g_2}^{g_1=1}\frac{k}{k_s}\frac{\rho - \rho_s^*}{\rho_s}\,\mathrm{d}g_1\right)\mathrm{d}g_2\right]\mathrm{d}g_3\right\}^{\frac{1}{4}} \tag{65}$$

and

$$\tilde{\eta}/\tilde{\phi} = \int_0^g \frac{k}{k_s}\,\mathrm{d}g \Big/ \int_0^1 \frac{k}{k_s}\,\mathrm{d}g. \tag{66}$$

Expressions (64) and (65) have, according to Labuntsov [9], been derived by Voskresenskiy [8].† The relevant integrals have been evaluated approximately by Labuntsov [9] on assuming that the density of the condensate is independent of the temperature and that the conductivity and viscosity are temperature dependent as follows:

$$k = k_w + g(k_s - k_w), \tag{67}$$

and

$$\frac{1}{\mu} = \frac{1}{\mu_w} + g\left(\frac{1}{\mu_s} - \frac{1}{\mu_w}\right). \tag{68}$$

Labuntsov obtains the result:

$$\epsilon(T_w;T_s) = \left(\frac{k_w^3\mu_s}{k_s^3\mu_w}\right)^{\frac{1}{4}} \times \left[\frac{30 + 85\dfrac{k_s}{k_w} + 76\dfrac{k_s^2}{k_w^2} + 19\dfrac{k_s^3}{k_w^3} + \dfrac{\mu_w}{\mu_s}\left(5 + 20\dfrac{k_s}{k_w} + 29\dfrac{k_s^2}{k_w^2} + 16\dfrac{k_s^3}{k_w^3}\right)}{280}\right]^{\frac{1}{4}} \tag{69}$$

and suggests, for $0{\cdot}5 \leqslant k_w/k_s \leqslant 2$ and $0{\cdot}1 \leqslant \mu_w/\mu_s \leqslant 1$, that expression (69) is adequately represented by the simpler result:

$$\epsilon(T_w;T_s) \simeq \left(\frac{k_w^3\mu_s}{k_s^3\mu_w}\right)^{\frac{1}{8}}. \tag{70}$$

Ignoring the negligibly small density factor on the right-hand side of (64) it is seen that the Voskresenskiy factor $\epsilon(T_w; T_s)$ gives the effect of variable physical properties of the condensate on flow and heat transfer. The alternative approach, of Drew [7], to account for the effect

---

† At present the authors have been unable to obtain a copy of the paper by Voskresenskiy [8].

of variable physical properties is to use Nusselt's formulae [see equations (46)] with the property values taken at an effective reference temperature. To derive this approximation Drew has assumed that ρ and k are independent of the temperature and μ varies inversely as the temperature as given by expression (68). Returning to (65) a simple integration yields the result:

$$\epsilon(T_w; T_s) = \left\{ \frac{k^3 \rho(\rho - \rho_s{}^*)}{k_s^3 \rho_s^2} \mu_s \left[\frac{1}{\mu_w} + 0{\cdot}25 \left(\frac{1}{\mu_s} - \frac{1}{\mu_w} \right) \right] \right\}^{\frac{1}{4}}. \quad (71a)$$

Thus the effect of variable viscosity is accounted for by evaluating the viscosity in the Nusselt formula at reference temperature T_D. Drew further assumes that ρ and k in (71a) are evaluated at this reference temperature. In a similar fashion the Voskresenskiy factor can be approximated by the result:

$$\epsilon(T_w; T_s) = \left\{ \frac{k^3 \mu_s(\rho - \rho_s^*)}{k_s^3 \mu \rho_s} \right\}_{M-S}^{\frac{1}{4}} \quad (71b)$$

where ρ, k and μ are evaluated at the effective reference temperature $T_{M-S} = T_w + 0{\cdot}31 (T_s - T_w)$, as deduced by Minkowycz and Sparrow [10].

In Table 4 the Voskresenskiy factor $\epsilon(T_w; T_s)$ is evaluated for saturated water using the Labuntsov expressions (69) and (70), the Drew expression (71a) and the Minkowycz and Sparrow expression (71b). Exact numerical values of the triple integral have been computed with the property relations (38) by representing the integral as three first order differential equations and solving these by Gill's modification of the Runge–Kutta method. The agreement between the exact values for $\epsilon(T_w, T_s)$ and those for the Labuntsov equation (69) is nearly precise. However, it should be remarked that ϵ depends on the fourth root of this integral and in general it does not follow that the simple property relations (67) and (68) should replace those given in (38). Moreover in the derivation of (69) ρ has been replaced by ρ_s and once again the fourth root helps to mask the small order effects due to density variations. Finally, the values of $\epsilon(T_w; T_s)$ as given by either the Drew expression (71a) or the Minkowycz and Sparrow expression (71b) are seen to be more accurate than those obtained using the Labuntsov expression (70). The latter result is not surprising since in [10] the reference temperature T_{M-S} has been deduced from numerical solutions of equations (53) to (55) using accurate values of ρ, μ and k with the temperature difference $(T_s - T_w)$ varying from 1 to 25°C.

DISCUSSION OF RESULTS FOR THE CONDENSATION OF STEAM

In the following sections the variable fluid property similarity solutions obtained for the laminar film condensation of steam are discussed in relation to the known constant fluid property solutions [1–6] and the approximate variable fluid property solutions [7–9].

Table 4

$T_s - T_w$	ρ, k, μ variable	Labuntsov, equation (69)	Labuntsov, equation (70)	Drew, equation (71a)	Minkowycz and Sparrow, equation (71b)
100	0·7202	0·7286	0·7324	0·7149	0·7270
90	0·7541	0·7577	0·7738	0·7467	0·7628
60	0·8492	0·8477	0·8753	0·8420	0·8574
30	0·9315	0·9297	0·9478	0·9268	0·9335
10	0·9813	0·9780	0·9845	0·9797	0·9828

Velocity and thermal profiles

Representative velocity and temperature functions are given in Table 3 for the condensation of steam at 100°C in the extreme case when the wall temperature $T_w = 0°C$. It is seen that the condensate velocity profile has nearly zero slope at the interface. In fact the shear stress changes sign at $\eta/\phi = 0{\cdot}98$. Note also that the vapour inflow velocity [as calculated using expression (32)] is virtually constant.

In Fig. 1 dimensionless velocity profiles $v_1/(4x_1g)^{\frac{1}{2}}$ are given for $T_w = 0°C$ and $T_w = 90°C$, respectively. Physically the liquid and vapour velocities are continuous at the interface, $\eta/\phi = 1$ or $\eta^* = 0$, and the apparent discontinuity shown graphically is due to the method of plotting. These profiles are compared with (a) the Nusselt profiles given by expression (45) with property values taken at effective reference temperatures $T_r = T_w + \beta(T_s - T_w)$ with $\beta = 0{\cdot}33$ for $T_w = 0°C$ and $\beta = 0{\cdot}26$ for $T_w = 90°C$, respectively,† and (b) the related Voskresenskiy profiles given by expression (62) and calculated using the variable property relations (38). The liquid phase velocity profiles due to Nusselt, Voskresenskiy and that of the present paper are identical when $T_w = 90°C$. In fact as $T_w \to T_s$ the combined effects of inertia forces, convective heat transfer, interfacial shear and variable physical properties are negligibly small. This result has already been established by Koh, Sparrow and Hartnett [5] in the stipulation that for a constant fluid property model these effects can be neglected if

$$Pr \geqslant 1 \text{ and } K = \frac{h_{fg}}{c_p(T_s - T_w)} \geqslant 5.$$

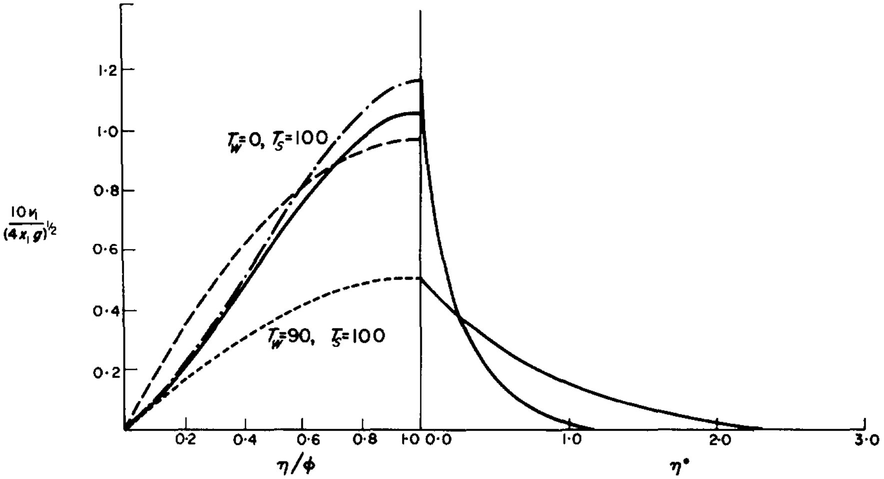

FIG. 1. Representative dimensionless velocity profiles for $T_w = 0$ and 90°C; Voskresenskiy —·—·—, Nusselt - - - - - - present calculation ————.

For $T_w = 0°C$ these combined effects are more apparent. At the interface the condensate velocity in the Voskresenskiy theory is +12 per cent in error. It follows, since inertia forces and convective terms are probably negligible, that this discrepancy is due to the neglect of vapour drag at the interface. It should be noted that for the velocity profile in the case $T_w = 0°C$, the close overall agreement between the present calculation and that due to Nusselt has been achieved using an effective reference temperature. With $\beta = 0{\cdot}33$ the condensate velocity at the interface is in error by $-6{\cdot}5$ per cent whilst using $\beta = 0{\cdot}31$, as proposed by Minkowycz and

---

† These values of the reference temperatures are obtained on correlation of wall heat-transfer coefficients obtained using Nusselt's constant fluid property model with the calculated variable fluid property results. The correlation procedure will be discussed later, see expression (81).

Sparrow [10], for correlation of heat-transfer results the error is −11 per cent.

The liquid phase thermal profile (see Table 3) is nearly linear indicating that heat-transfer by conduction is dominant. For example, in the case $T_w = 0°C$, the maximum deviation from the linear profile is only +6·5 per cent occurring at approximately $\eta/\phi = 0·4$. Such deviations are probably due to the effect of variable conductivity rather than to the inclusion of convective heat-transfer etc., in the governing equations. In this connexion it is interesting to point out that the dimensionless temperature function G, (as given in Table 3) when rounded off to three decimal places, is identical to the dimensionless conduction profile in fully developed Couette flow of water when the fixed plate is maintained at a temperature 0°C and the moving plate at a temperature 100°C (see [14]).

FILM THICKNESS AND CONDENSATION RATE

The condensate film thickness is determined by expression (24); the integral

$$\int_0^{\phi} \frac{\rho_s}{\rho}\,\mathrm{d}\eta$$

is tabulated in Table 2 for various $\Delta T = T_s - T_w$. It is convenient to consider the dimensionless ratio:

$$\frac{\delta_{x_1}}{(\delta^N_{x_1})_s} = \left\{\left(1 - \frac{\rho_s{}^*}{\rho_s}\right) Pr_s\,K_s\right\}^{\frac{1}{4}} \int_0^{\phi} \frac{\rho_s}{\rho}\,\mathrm{d}\eta, \qquad (72)$$

where $(\delta^N_{x_1})_s$ is the Nusselt film thickness defined by equation (46a) and evaluated at the temperature $T_s = 100°C$. Similarly for the condensation rate the relevant dimensionless ratio is

$$\frac{\Gamma_{x_1}}{(\Gamma^N_{x_1})_s} = 3\left(1 - \frac{\rho_s{}^*}{\rho_s}\right)^{-\frac{1}{4}} (Pr_s\,K_s)^{\frac{3}{4}}\,F(\phi); \qquad (73)$$

$F(\phi)$ is tabulated in Table 2 for various ΔT. The dimensionless ratios $\delta_{x_1}/(\delta^N_{x_1})_s$ and $\Gamma_{x_1}/(\Gamma^N_{x_1})_s$ are given in Table 5 together with the percentage deviation of the Voskresenskiy formulae [equations (63, 65, 38) for $\delta_{x_1}/(\delta^N_{x_1})_s$ and (64, 65, 38) for $\Gamma_{x_1}/(\Gamma^N_{x_1})_s$] and the approximate Labuntsov formula [equations (64) and (70) for $\Gamma_{x_1}/(\Gamma^N_{x_1})_s$], respectively. Obviously the neglect of interfacial shear in the Voskresenskiy theory produces the observed small over-estimation in the dimensionless film thicknesses and condensation rates. In the extreme case $T_w = 0°C$ the percentage deviation of the Voskresenskiy formulae for the dimensionless condensation rate is +2·5.

It is interesting to compare this latter result with that obtained from the constant fluid property model developed by Chen [see expression (48b)]. Using the Drew reference temperature T_D the dimensionless condensation rate $\Gamma^C_{x_1}/(\Gamma^N_{x_1})_s$ is in error by at most −4·4 per cent when $T_w = 0°C$. Further calculation yields the effective reference temperature to be used in conjunction with the Chen expression (48b). If

$$T_r = T_w + \alpha^C(T_s - T_w), \qquad (74)$$

and

$$\alpha^C = \alpha^C_0 + \alpha^C_1\left(\frac{T_s - T_w}{T_s}\right) + \alpha^C_2\left(\frac{T_s - T_w}{T_s}\right)^2. \qquad (75)$$

Table 5

$T_s - T_w$	$\frac{\delta_{x_1}}{(\delta^N_{x_1})_s}$	Per cent deviation of Voskresenskiy	$\frac{\Gamma_{x_1}}{(\Gamma^N_{x_1})_s}$	Per cent deviation of Voskresenskiy	Per cent deviation of Labuntsov
100	1·271	1·4	0·7028	2·5	4·2
90	1·235	1·1	0·7356	2·5	5·2
60	1·138	0·6	0·8297	2·4	5·5
30	1·064	0·04	0·9184	1·4	3·2
10	1·023	−0·10	0·9780	0·4	0·7

the method of least squares gives the "best-fit" for the Chen expression as

$$\frac{(\Gamma_{x_1})_r}{(\Gamma^N_{x_1})_s} = \frac{\Gamma_{x_1}}{(\Gamma^N_{x_1})_s} \qquad (76)$$

with $\alpha_0^C = 0{\cdot}2959$, $\alpha_1^C = -0{\cdot}1437$ and $\alpha_2^C = 0{\cdot}1646$, respectively.

HEAT-TRANSFER RESULTS

The characteristic quantities representing the heat transfer at the wall are h_{x_1} and Nu_{x_1} as defined in equation (41). In terms of the dimensionless variables (20) and the Nusselt expression (46c) the dimensionless ratio involving the local wall heat-transfer coefficient is

$$\frac{h_{x_1}}{(h^N_{x_1})_s} = \frac{\rho_w k_w}{\rho_s k_s}\left[\left(1 - \frac{\rho_s^*}{\rho_s}\right) Pr_s\, K_s\right]^{-\frac{1}{4}} \left(\frac{\mathrm{d}G}{\mathrm{d}\eta}\right)_{\eta=0} \qquad (77)$$

and similarly for the local Nusselt number

$$\frac{Nu_{x_1}}{(Nu^N_{x_1})_s} = \frac{k_s}{k_w}\frac{h_{x_1}}{(h^N_{x_1})_s}; \qquad (78)$$

values of $(\mathrm{d}G/\mathrm{d}\eta)_0$ are given in Table 2 for various ΔT. The dimensionless ratios $h_{x_1}/(h^N_{x_1})_s$ and $Nu_{x_1}/(Nu^N_{x_1})_s$ are given in Table 6 together with the relevant percentage deviations of the Voskresenskiy formula [equations (64, 65, 38)] and the approximate Labuntsov formula [equations (64) and (70)]. It is seen that the Voskresenskiy theory yields maximum underestimates of −3·3 and −3·4 per cent in $h_{x_1}/(h^N_{x_1})_s$, respectively, for the extreme case $T_w = 0$°C. Due to vapour drag the actual film thickness is less than that calculated from the Voskresenskiy theory. The actual local heat-transfer coefficient must then increase accordingly.

Table 6

$T_s - T_w$	$\frac{h_{x_1}}{(h^N_{x_1})_s}$	Per cent deviation		$\frac{Nu_{x_1}}{(Nu^N_{x_1})_s}$	Per cent deviation	
		Voskresenskiy	Labuntsov		Voskresenskiy	Labuntsov
100	0·7448	−3·3	−2·2	0·9233	−3·4	−1·7
90	0·7762	−2·8	−2·4	0·9220	−2·8	−0·3
60	0·8631	−1·6	−1·8	0·9298	−1·6	1·4
30	0·9376	−0·65	−0·8	0·9584	−0·60	1·1
10	0·9826	−0·13	−0·45	0·9870	−0·12	0·21

The alternative approach to allow for the effect of variable fluid properties is now examined. Using the Drew reference temperature T_D the dimensionless ratios $(h^N_{x_1})_D/(h^N_{x_1})_s$, $(h^R_{x_1})_D/(h^N_{x_1})_s$, and $(h^C_{x_1})_D/(h^N_{x_1})_s$ are in error by −5·8, −2·9 and −3·5 per cent, respectively, when $T_w = 0$°C. In this extreme case the corresponding error in $(h^N_{x_1})/(h^N_{x_1})_s$ using the Minkowycz and Sparrow reference temperature T_{M-S} is −1·4 per cent. This close agreement is due to the judicious choice of the reference temperature T_{M-S}, since the actual error in the Voskresenskiy theory is −3·3 per cent. Precise agreement between the various expressions for the heat-transfer coefficients due to Nusselt, Rohsenow and Chen, respectively, and those obtained on the present paper can be achieved by evaluating all the properties appearing in these expressions at the reference temperatures:

$$T_r = T_w + \beta(T_s - T_w), \qquad (79)$$

where

$$\beta = \beta_0 + \beta_1\left(\frac{\Delta T}{T_s}\right) + \beta_2\left(\frac{\Delta T}{T_s}\right)^2. \qquad (80)$$

The method of least squares was employed to obtain the coefficients β_0, β_1 and β_2, so that

$$\frac{h_{x_1}}{(h^N_{x_1})_s} = \frac{(h^N_{x_1})_r}{(h^N_{x_1})_s} \qquad (81)$$

for the Nusselt formula and similar expressions for the Rohsenow and Chen formula. These calculations have been repeated to give correlations of the form:

$$\frac{Nu_{x_1}}{(Nu^N_{x_1})_s} = \frac{(Nu^N_{x_1})_r}{(Nu^N_{x_1})_s}. \qquad (82)$$

The least squares estimates of the coefficients β_0, β_1 and β_2 are listed in Table 7. It is proposed that the above reference temperatures are valid for the complete range of wall temperatures $T_w = 0$–$100°C$. For example, in the evaluation of the rates $(h^N_{x_1})_r/(h^N_{x_1})_s$ the values for the coefficient β are 0·33, 0·32$_5$, 0·31, 0·30 and 0·26 when $T_s - T_w = 100$, 70, 40, 30 and 10°C, respectively. Thus for small temperature differences of order 10°C the Drew coefficient $\beta = 0{\cdot}25$ can be adopted as this gives a maximum error of less than 0·03 per cent. Even for $\Delta T \leqslant 30°C$ the maximum possible error using the Drew coefficient is 0·5 per cent whilst the Minkowycz and Sparrow coefficient $\beta = 0{\cdot}31$ yields a maximum error of 0·2 per cent.

Table 7

	$h_{x_1}/(h^N_{x_1})_s$			$Nu_{x_1}/(Nu^N_{x_1})_s$		
	β_0	β_1	β_2	β_0	β_1	β_2
Nusselt	0·2474	0·1580	−0·0769	0·5371	−0·4819	0·6478
Rohsenow	0·1602	0·1649	−0·0356	0·3490	−0·2323	0·4832
Chen	0·2228	0·1003	−0·0255	0·4852	−0·4845	0·6448

CONCLUDING REMARKS

It has been shown, from the variable fluid property similarity solutions, evaluated for the laminar film condensation of steam, the wall heat-transfer coefficients, obtained using Nusselt's constant fluid property model with Drew reference temperature, are in error by at most −5·8 per cent. This discrepancy is due to the neglect of the non-linear effects of variable physical properties in the condensate and the effect of vapour drag; effects due to the omission of the inertia forces and convective heat transfer are unimportant. However, experimental data on heat-transfer coefficients, according to McAdams [7], are at most 28 per cent higher than those deduced from Nusselt's theory using the Drew reference temperature. The resulting discrepancy, of order 22 per cent, between experiment and the present results is known to be caused by surface tension. Kapitsa [17] has shown that for a critical Reynolds number $Re_{x_1} = 4\Gamma_{x_1}/\mu_s \geqslant 33$ waves begin to form on the surface of the laminar film. Kapitsa has also shown that the average film thickness is reduced, thus leading to a corresponding increase in the heat-transfer coefficient. In practice the effect of ripple formation on the surface of the laminar film is accounted for by using a correction factor of 1·2. Further references dealing with the correlation of experimental data, to account for surface tension effects, are given by Kutateladze [18] and in the review paper by Chisholm and Provan [19].

REFERENCES

1. W. NUSSELT, Die Oberflachen Kondensation de Wasserdampfes, *Z. Ver. D. Ing.* **60**, 541, 569 (1916).
2. L. A. BROMLEY, Effect of heat capacity of condensate in condensing, *Ind. Engng Chem.* **44**, 2966–2969 (1952).
3. W. M. ROHSENOW, Heat transfer and temperature distribution in laminar film condensation, *Trans. Am. Soc. Mech. Engrs* **78**, 1645–1648 (1956).
4. E. M. SPARROW and J. L. GREGG, A boundary-layer treatment of laminar film condensation, *J. Heat Transfer* **81**, 13–18 (1959).
5. J. C. Y. KOH, E. M. SPARROW and J. P. HARTNETT, The two phase boundary layer in laminar film condensation, *Int. J. Heat Mass Transfer* **2**, 69–82 (1961).
6. M. M. CHEN, An analytical study of laminar film condensation: Part 1—Flat plates, *J. Heat Transfer* **81**, 48–54 (1961).
7. W. H. MCADAMS, *Heat Transmission*, 3rd edn. McGraw-Hill, New York (1954).

8. K. D. Voskresenskiy, Calculation of heat transfer in film condensation allowing for the temperature dependence of the physical properties of the condensate, U.S.S.R. Acad. Sci., OTK (1948).
9. D. A. Labuntsov, Effect of temperature dependence of physical parameters of condensate on heat transfer in film condensation of steam, *Teploenergetika* **4**(2), 49–51 (1957).
10. W. J. Minkowycz and E. M. Sparrow, Condensation heat transfer in the presence of non-condensables, interfacial resistance, super heating, variable properties, and diffusion, *Int. J. Heat Mass Transfer* **9**, 1125–1144 (1966).
11. E. R. G. Eckert and R. M. Drake Jr., *Heat and Mass Transfer*, 2nd edn. McGraw-Hill, New York (1959).
12. S. Goldstein, *Modern Developments in Fluid Dynamics*, Vol. 1, p. 119. Clarendon Press, Oxford (1938).
13. Y. R. Mayhew and G. F. C. Rogers, *Thermodynamic and Transport Properties of Fluids*. Blackwell, Oxford (1964).
14. G. Poots and M. H. Rogers, Laminar flow between parallel flat plates, with heat transfer, of water with variable physical properties, *Int. J. Heat Mass Transfer* **8**, 1515–35 (1965).
15. R. A. Buckingham, *Numerical Methods*, Chapter 10. Pitman, London (1962).
16. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd edn, p. 11. Clarendon Press, Oxford (1959).
17. P. L. Kapitsa, Wave flow of thin layers of a viscous fluid, *Zh. Eksp. Teoret. Fiz.* **18**, 1 (1948).
18. S. S. Kutateladze, *Fundamentals of Heat Transfer*, pp. 298–326. Edward Arnold, London (1963).
19. D. Chisholm and T. F. Provan, A review of heat and mass transfer during condensation in surface heat exchanges, N.E.L. Report 96 (1963).

Résumé—Une étude théorique a été faite sur la condensation en régime permanent par film laminaire bidimensionnel de vapeur d'eau saturée pure à 100°C et à pression atmosphérique sur une paroi verticale isotherme. Les propriétés physiques variables du condensat sont supposées être celles de l'eau saturée aux températures convenables. En négligeant les effets de la tension superficielle à l'interface liquide–vapeur, les équations de base pour les phases liquide et vapeur sont simplifiées en employant les approximations de la couche limite et elles admettent une solution en fonction des variables de similitude. A partir des solutions numériques des équations différentielles ordinaires qui en découlent, les caractéristiques de l'écoulement pour les phases liquide et vapeur sont déterminées pour des températures de paroi de 0, 10, 40, 70 et 90°C. Les résultats correspondant au transport de chaleur à la paroi et au flux de masse dans la phase liquide sont présentés et comparés en détail avec ceux des théories existantes.

Zusammenfassung—Eine theoretische Untersuchung wurde durchgeführt für die stationäre, zweidimensionale, laminare Filmkondensation von reinem gesättigten Dampf bei 100°C und Atmosphärendruck an einer isothermen senkrechten Wand. Als die veränderlichen Stoffwerte des Kondensats werden jene des gesättigten Wassers bei der entsprechenden Temperatur verwendet. Bei Vernachlässigung der Einflüsse der Oberflächenspannung an der Flüssigkeits- und Dampfgrenzfläche lassen sich die Bestimmungsgleichungen für die flüssige und gasförmige Phase durch Grenzschichtnäherungen vereinfachen und ergeben eine Lösung in Form von Ähnlichkeitsvariablen. Aus numerischen Lösungen der sich ergebenden gewöhnlichen Differentialgleichungen werden Strömungskomponenten für die flüssige und die dampfförmige Phase für Wandtemperaturen von 0, 10, 40, 70 und 90°C bestimmt. Die zugehörigen Ergebnisse für den Wärmeübergang und den Massenstrom in der flüssigen Phase werden angegeben und im einzelnen mit Werten anderer Theorien verglichen.

Аннотация—Проведено теоретическое исследование установившейся двумерной ламинарной пленочоной конденсации чистого насыщенного пара при 100°C и атмосферном давлении на изотермической вертикальной стенке. За переменные физические свойства жидкого конденсата взяты физические свойства насыщенной воды при соответствующих температурах. Пренебрегая влиянием поверхностного натяжения на границе раздела фаз жидкость-пар можно упростить основные уравнения для жидкой и паровой фаз путем использования приближений пограничного слоя и получить решение в переменных, позволяющих свести задачу к автомодельной. На основе численных решений полученных обычных дифференциальных уравнений можно определить характеристики потока для жидкой и паровой фаз при температурах стенки 0, 10, 40, 70 и 90°C. Полученные результаты для теплообмена на стенке и потока массы в жидкой фазе детально сравнивались с результатами существующих теорий.